KATHOLIEKE UNIVERSITEIT
LEUVEN

Divertor Design through Shape Optimization

W. Dekeyser[1], D. Reiter[2], M. Baelmans[1]

[1] Department of Mechanical Engineering, Katholieke Universiteit Leuven, Belgium

[2] Institute of Plasma Physics, Forschungszentrum Jülich GmbH, EURATOM Association, TEC, Germany

Overview

- Introduction
- Target design for homogeneous power load
- Test problem
- Conclusion

Introduction

- Heating power of next generation fusion reactors

Device name Divertor: SD/XD	Heating power P (MW)	Major radius R (m)	P_{heat}/R ITER=1
C-Mod	3	0.6	0.26
DIII-D	10	1.6	0.31
JET	17	3	0.31
JT-60U	17	3.4	0.26
ITER	120	6.2	1
EU-A	1246	9.6	6.8
EU-B	990	8.6	6.1
EU-C	792	7.5	5.6
EU-D	571	6.1	4.9
ARIES-AT	387	5.2	3.9
ARIES-RS	515	5.5	4.9
Slim-CS	645	5.5	6.2
CREST	691	5.4	6.7

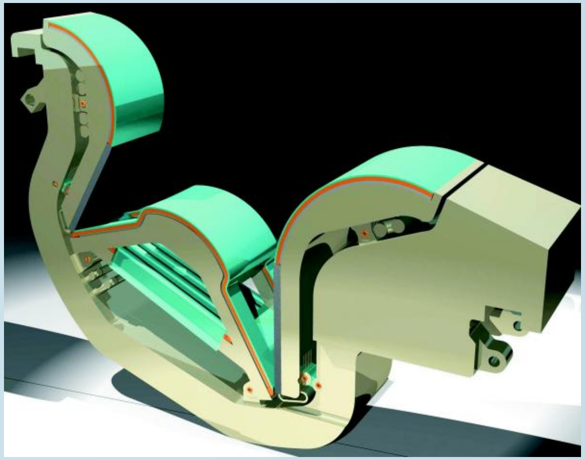
(Kotschenreuther et al., Phys. Plasmas 14, 072502 (2007).)

Challenging divertor design problem

KATHOLIEKE UNIVERSITEIT LEUVEN

Large number of design variables
(Parameterized) shape of divertor, currents through divertor coils,…

Complex physical model
Time consuming simulations
Fluid plasma model (e.g. B2)
kinetic neutrals (e.g. EIRENE)


http://www.iter.org

Physics, material and engineering constraints
E.g. core stability, peak heat flux limits, neutron shielding,…

Divertor design in an optimization framework

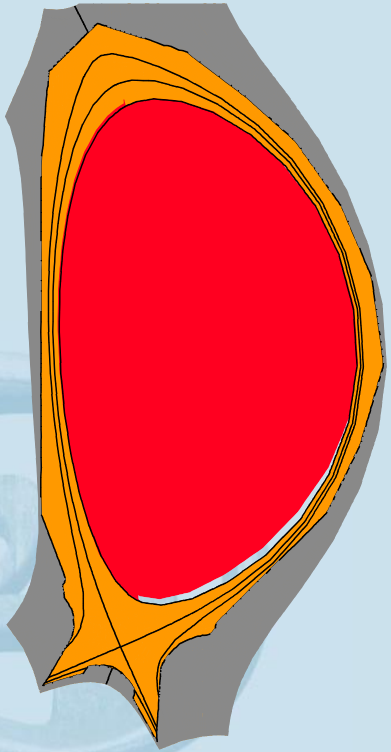
- Incorporate design requirements in a cost functional
- Find the optimal design using gradient based optimization algorithms
- Advantages:
  - **Automated** design process
  - **Efficient** solution using advanced adjoint methods
    - computational time independent of number of design variables!
  - Natural framework to include various design **constraints**

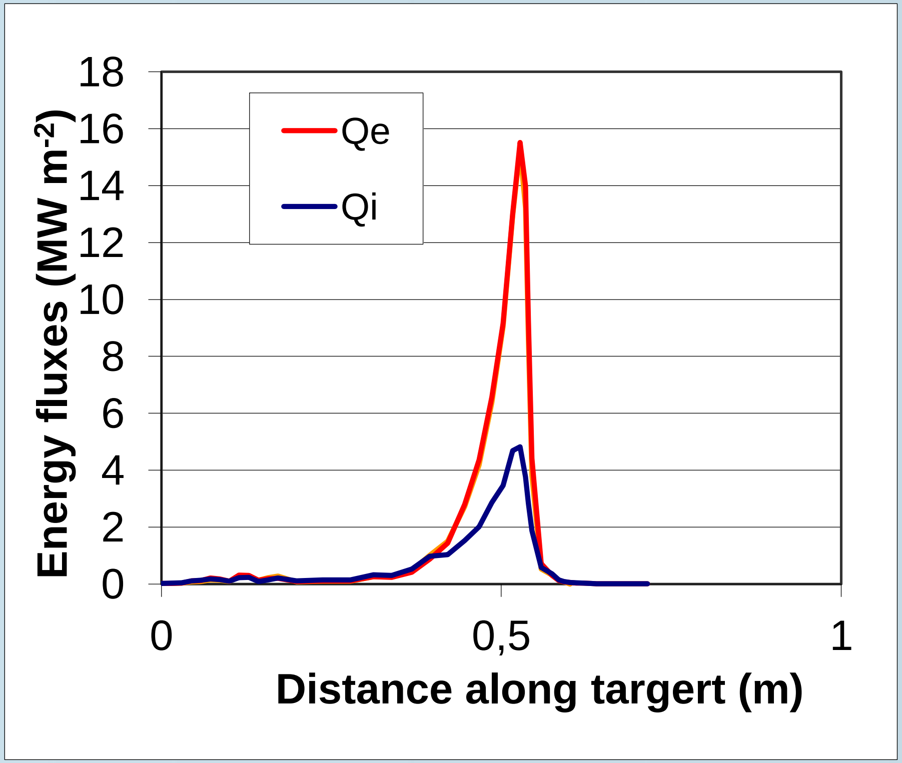
Overview

- Introduction
- Target design for homogeneous power load
- Test problem
- Conclusion

Motivation

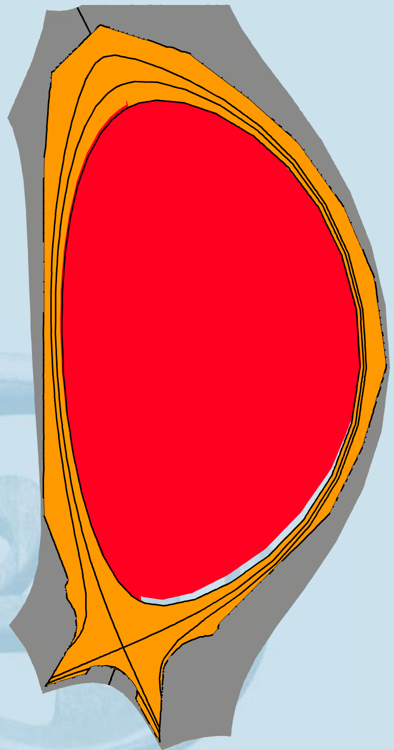
- Energy fluxes in F12 ITER geometry
  - Energy fluxes strongly peaked
  - > 10 MW m^{-2}

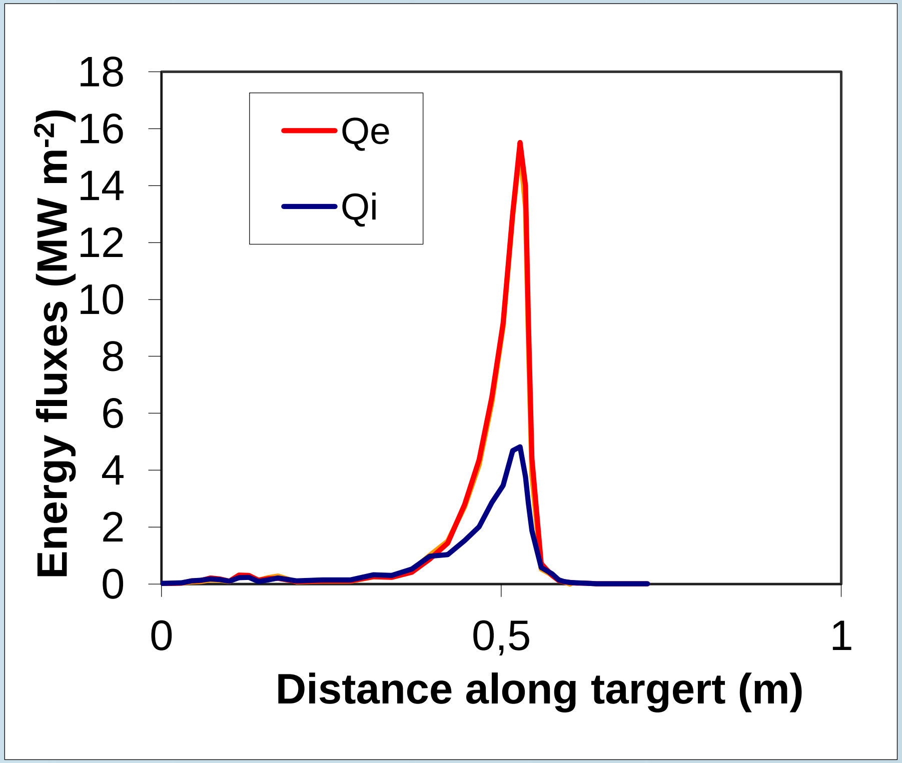




Cost functional

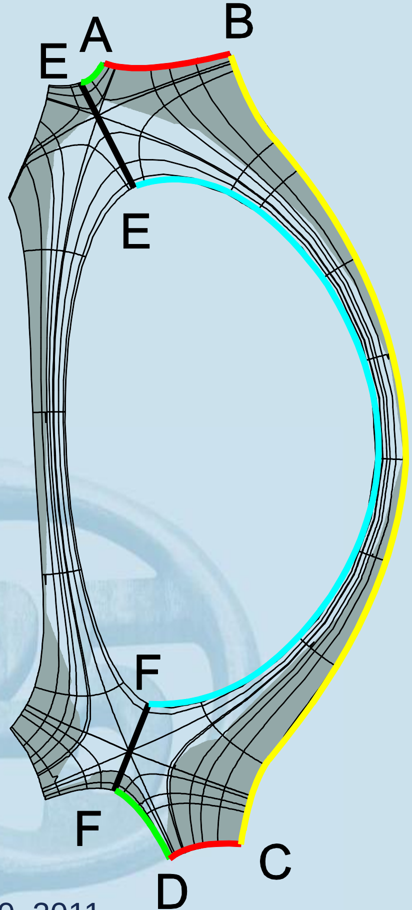
$$J(\mathbf{q}, \phi) = \frac{1}{2} \int_{\mathrm{T}(\phi)} (Q - Q_{\mathrm{d}})^2 \, \mathrm{d}\sigma$$

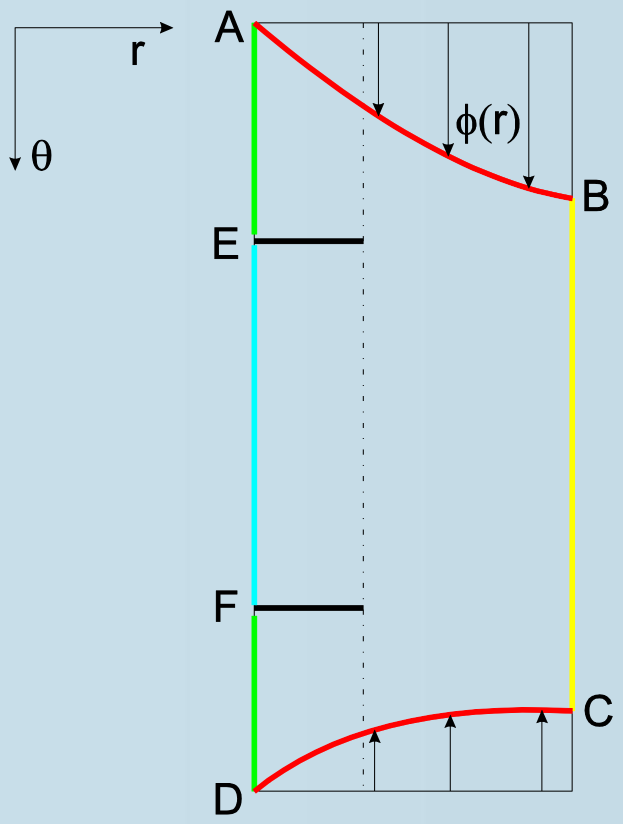




Shape parameterization

$$J(\mathbf{q}, \phi) = \frac{1}{2} \int_{\mathrm{T}(\phi)} (Q - Q_\mathrm{d})^2 \, \mathrm{d}\sigma$$





Model equations

- State equations for $\mathbf{q} = (n_\mathrm{i}, u_{||}, T, p_\mathrm{n})^\mathrm{T}$:

$$B_\Omega(\mathbf{q}) = \frac{1}{\sqrt{g}}\frac{\partial}{\partial\theta}\left(\frac{\sqrt{g}}{h_\theta}C(\mathbf{q}) - \frac{\sqrt{g}}{h_\theta^2}D^\theta(\mathbf{q})\frac{\partial\mathbf{q}}{\partial\theta}\right) - \frac{1}{\sqrt{g}}\frac{\partial}{\partial r}\left(\frac{\sqrt{g}}{h_r^2}D^r(\mathbf{q})\frac{\partial\mathbf{q}}{\partial r}\right) - S(\mathbf{q}) = 0$$

with

$$S_\mathrm{n} = \begin{pmatrix} n_\mathrm{i}n_\mathrm{n}\langle\sigma v\rangle_\mathrm{i} - n_\mathrm{i}^2\langle\sigma v\rangle_\mathrm{r} \\ -mn_\mathrm{i}^2(\langle\sigma v\rangle_\mathrm{r} + \langle\sigma v\rangle_\mathrm{c})u_{||} \\ -E_\mathrm{i}n_\mathrm{i}n_\mathrm{n}\langle\sigma v\rangle_\mathrm{i} \\ n_\mathrm{i}^2\langle\sigma v\rangle_\mathrm{r} - n_\mathrm{i}n_\mathrm{n}\langle\sigma v\rangle_\mathrm{i} \end{pmatrix}, \quad S_z = \begin{pmatrix} 0 \\ 0 \\ -c_z n_\mathrm{i}^2 L_z(T) \\ 0 \end{pmatrix}, \quad S_p = \begin{pmatrix} 0 \\ -\frac{b_\theta}{h_\theta}\frac{\partial p}{\partial\theta} \\ 0 \\ 0 \end{pmatrix}$$

Analytical approximations for ionization, recombination, and CX rates and radiative loss function (Carbon)

Optimization problem

- Cost functional:

$$J(\mathbf{q},\phi) = \frac{1}{2}\int_{\mathrm{T}(\phi)} (Q - Q_{\mathrm{d}})^2 \,\mathrm{d}\sigma$$

- Lagrangian function to enforce state equations constraints:

$$L(\mathbf{q},\phi,\mathbf{q}^*) = \int_{\mathrm{T}(\phi)} J_\sigma(\mathbf{q},\phi)\mathrm{d}\sigma - \int_{\mathrm{V}(\phi)} (\mathbf{q}^*_\Omega)^{\mathrm{T}} B_\Omega(\mathbf{q},\phi)\mathrm{d}\Omega - \int_{\mathrm{S}(\phi)} (\mathbf{q}^*_\sigma)^{\mathrm{T}} B_\sigma(\mathbf{q},\phi)\mathrm{d}\sigma$$

- Optimality conditions:

$$\begin{cases} L_{\mathbf{q}^*}(\mathbf{q}(\phi),\phi,\mathbf{q}^*) & = & 0 & \text{state equations} \\ L_{\mathbf{q}}(\mathbf{q}(\phi),\phi,\mathbf{q}^*) & = & 0 & \text{adjoint equations} \\ L_{\phi}(\mathbf{q}(\phi),\phi,\mathbf{q}^*) & = & 0 & \text{design equation} \end{cases}$$

Adjoint equations and gradient of cost functional

- Adjoint equations

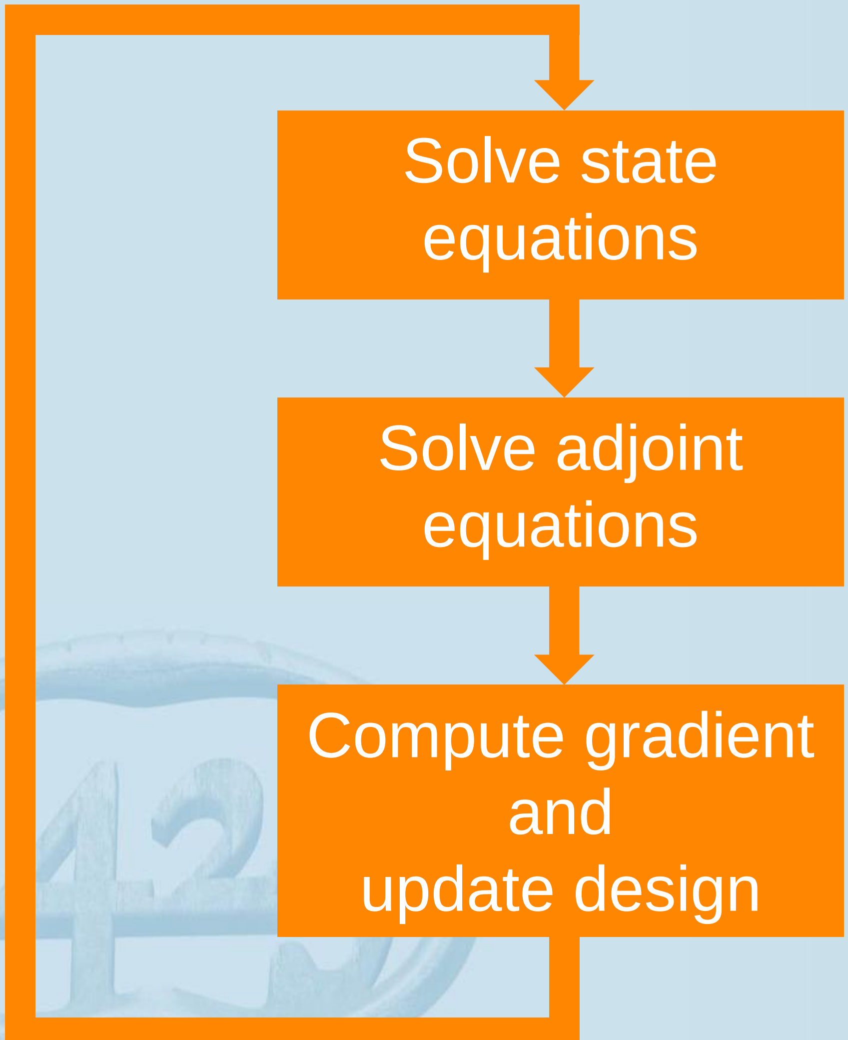
$$
\begin{aligned}
0 \quad = \quad & -C_{\mathbf{q}}^{\mathrm{T}} \frac{1}{h_\theta} \frac{\partial \mathbf{q}_\Omega^*}{\partial \theta} - \frac{1}{\sqrt{g}} \frac{\partial}{\partial \theta} \left(\frac{\sqrt{g}}{h_\theta^2} D^\theta \frac{\partial \mathbf{q}_\Omega^*}{\partial \theta} \right) - \frac{1}{\sqrt{g}} \frac{\partial}{\partial r} \left(\frac{\sqrt{g}}{h_r^2} D^r \frac{\partial \mathbf{q}_\Omega^*}{\partial r} \right) \\
& + \frac{1}{h_\theta} \frac{\partial \mathbf{q}^{\mathrm{T}}}{\partial \theta} \left(D_{\mathbf{q}}^\theta \right)^{\mathrm{T}} \frac{1}{h_\theta} \frac{\partial \mathbf{q}_\Omega^*}{\partial \theta} + \frac{1}{h_r} \frac{\partial \mathbf{q}^{\mathrm{T}}}{\partial r} \left(D_{\mathbf{q}}^r \right)^{\mathrm{T}} \frac{\partial \mathbf{q}_\Omega^*}{\partial r} - S_{\mathbf{q}}^* \mathbf{q}_\Omega^*
\end{aligned}
$$

  – Reversed characteristics!

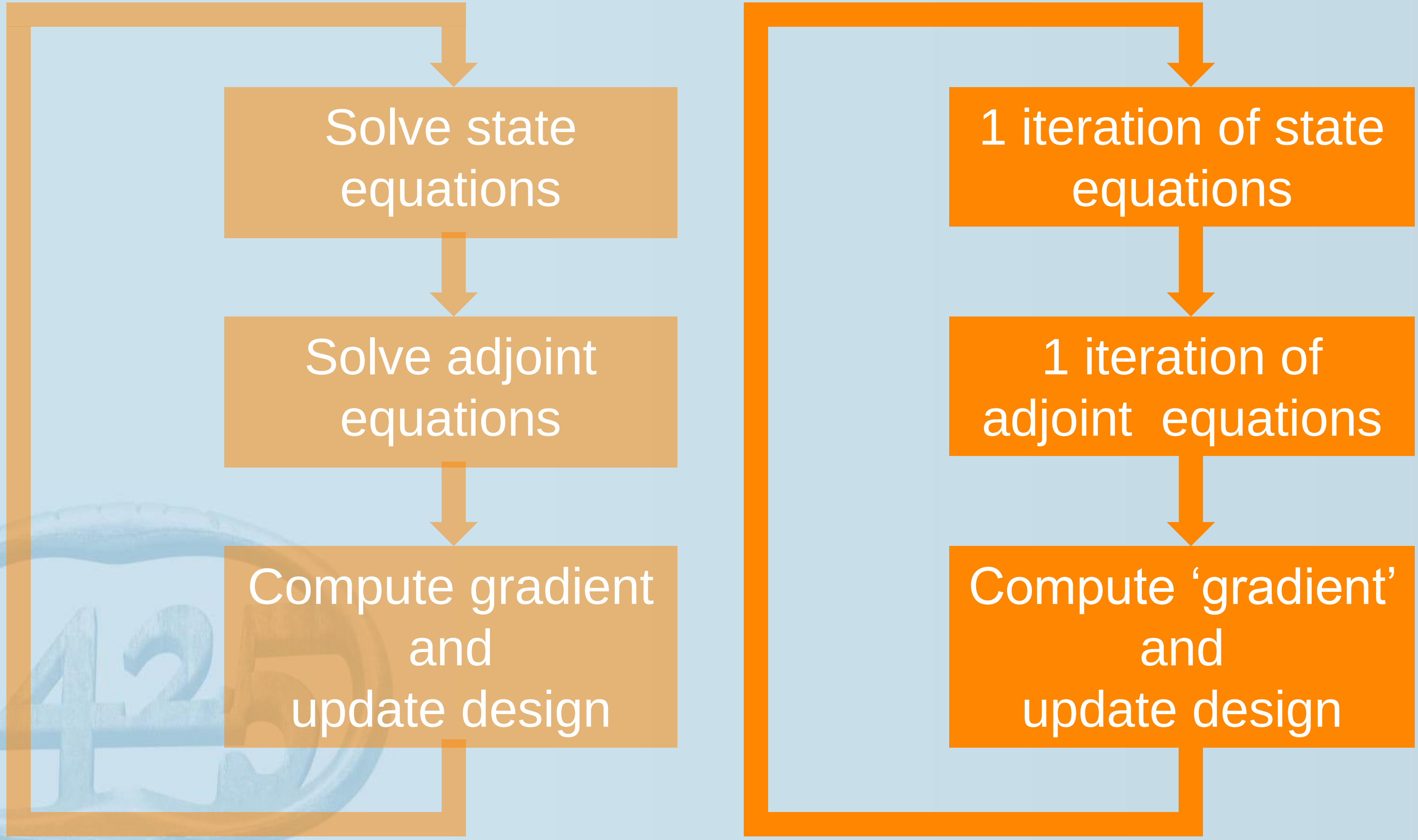
- Gradient of cost functional
  – Velocity method for limits of integration

$$
\begin{aligned}
\hat{J}'(\phi)\delta\phi \quad = \quad & L_\phi(\mathbf{q}(\phi), \phi, \mathbf{q}^*(\phi))\delta\phi \\
= \quad & \int_{\mathrm{T}(\phi)} \left(J_{\sigma,\phi}(\mathbf{q}, \phi)\delta\phi + \nabla J_\sigma(\mathbf{q}, \phi) \cdot \mathcal{V} + J_\sigma(\mathbf{q}, \phi) \left(\nabla \cdot \mathcal{V} - D\mathcal{V}\nu \cdot \nu \right) \right) \mathrm{d}\sigma - \\
& \int_{\mathrm{V}(\phi)} \left(\mathbf{q}_\Omega^* \right)^{\mathrm{T}} B_{\Omega,\phi}(\mathbf{q}, \phi)\delta\phi \mathrm{d}\Omega - \int_{\mathrm{S}(\phi)} \left(\mathbf{q}_\Omega^* \right)^{\mathrm{T}} B_\Omega(\mathbf{q}, \phi)\mathcal{V} \cdot \nu \mathrm{d}\sigma - \\
& \int_{\mathrm{S}(\phi)} \left(\mathbf{q}_\sigma^* \right)^{\mathrm{T}} \left(B_{\sigma,\phi}(\mathbf{q}, \phi)\delta\phi + \nabla B_\sigma(\mathbf{q}, \phi) \cdot \mathcal{V} + B_\sigma(\mathbf{q}, \phi) \left(\nabla \cdot \mathcal{V} - D\mathcal{V}\nu \cdot \nu \right) \right) \mathrm{d}\sigma
\end{aligned}
$$

Solution algorithm



Solution algorithm: *one-shot method*

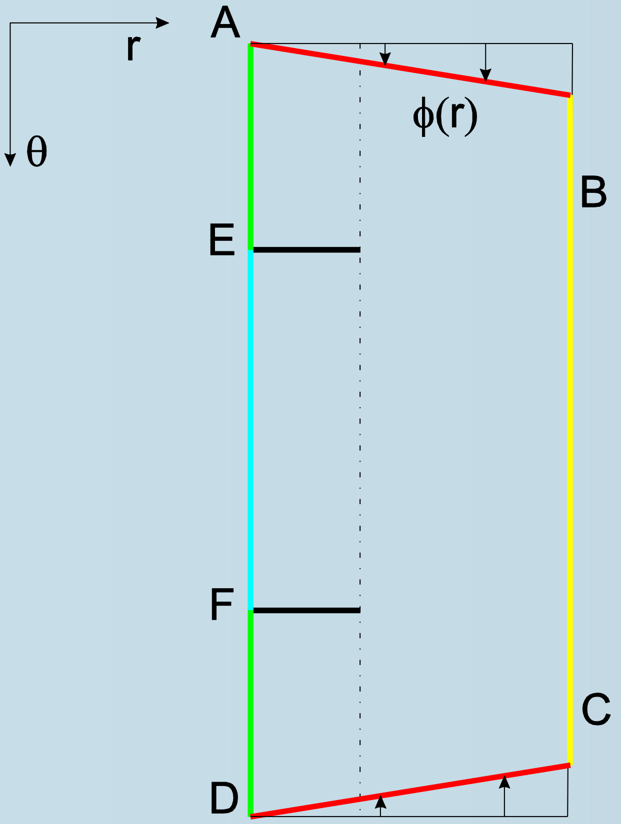


Overview

- Introduction
- Target design for homogeneous power load
- Test problem
- Conclusion

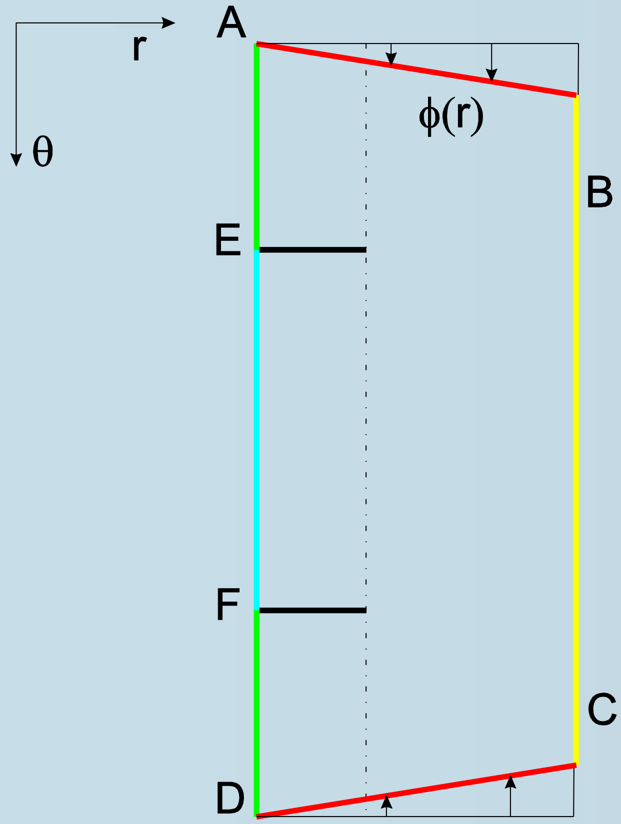
Test problem

- Outboard half of up-down symmetric connected double null divertor, modeled as cylindrical shell
- No flux expansion,…
- ITER-like parameters
  - Geometry:
    - Major radius 6 m
    - Core length 10 m
    - SOL width 0.1 m
  - Core density $2 \cdot 10^{19}$ m^{-3}
  - 50 MW input power from core
  - 30 m^{3} s^{-1} thermal D_2 pumping speed

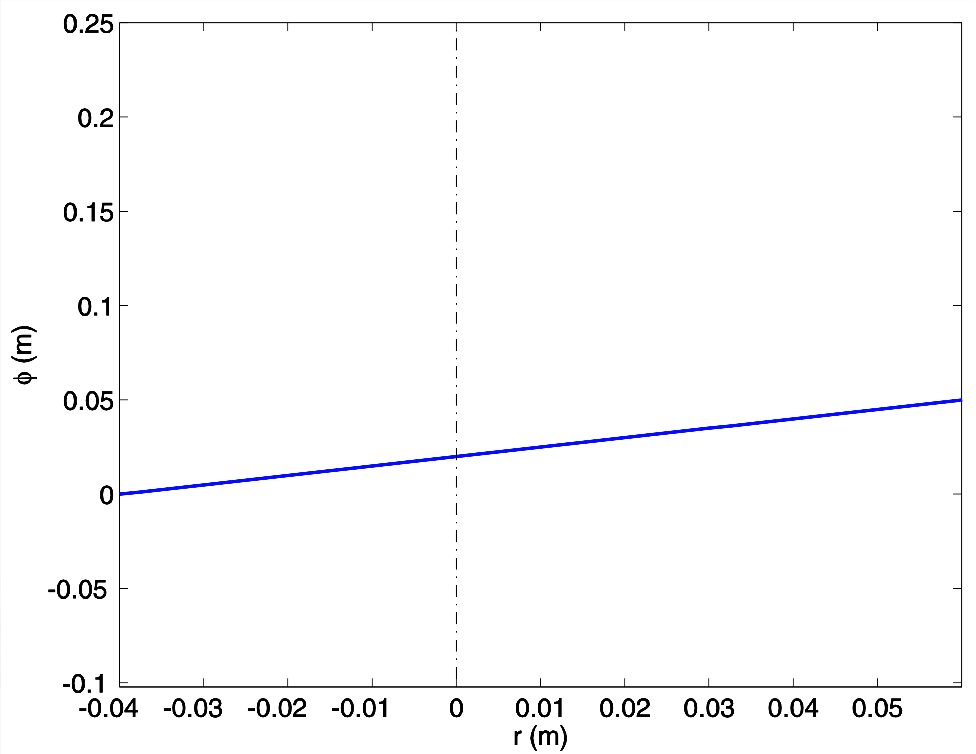


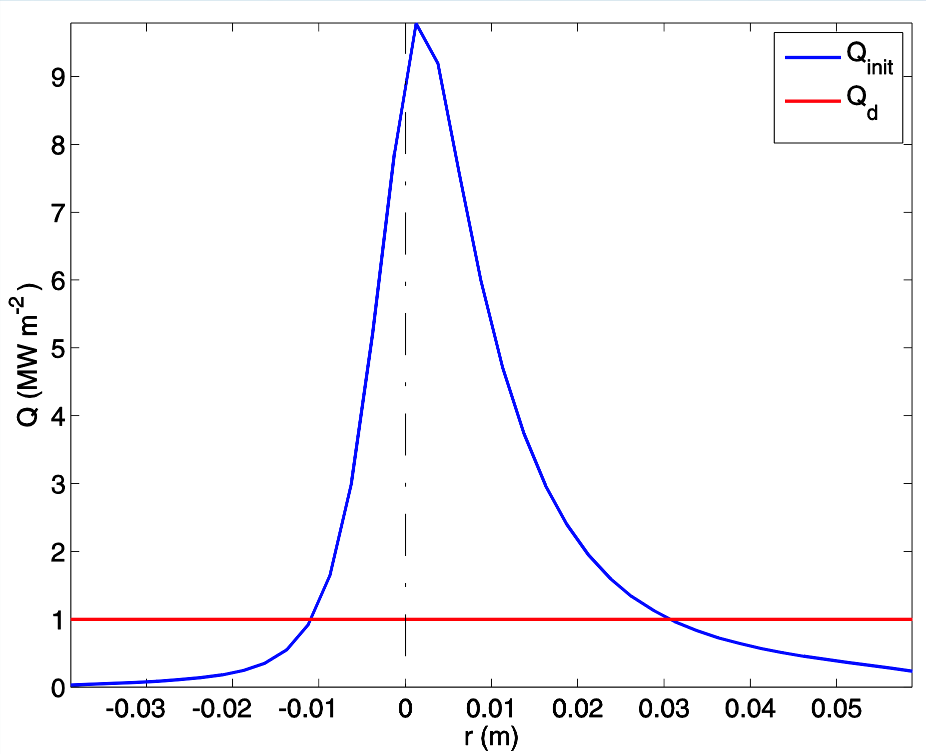
Grid and solver

- 130x40 grid
  - 80 design variables
- Poloidal grid lines aligned with magnetic field
- Radial grid lines deformed to match target surface
  - Explicit 9-point stencil for correct discretization of fluxes
- One-shot optimization algorithm

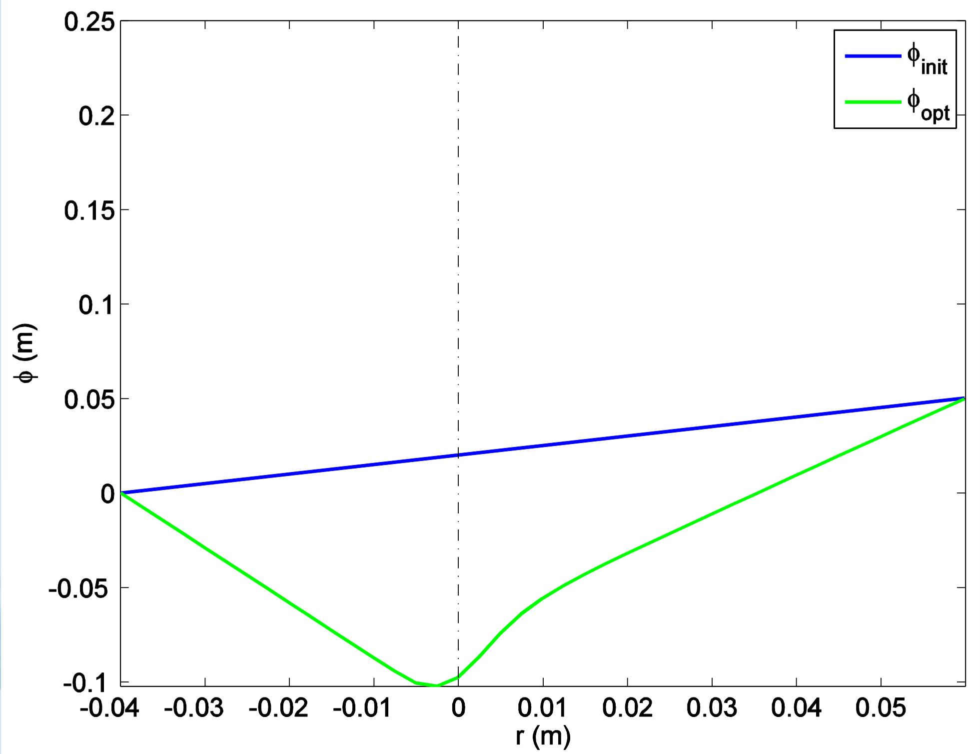


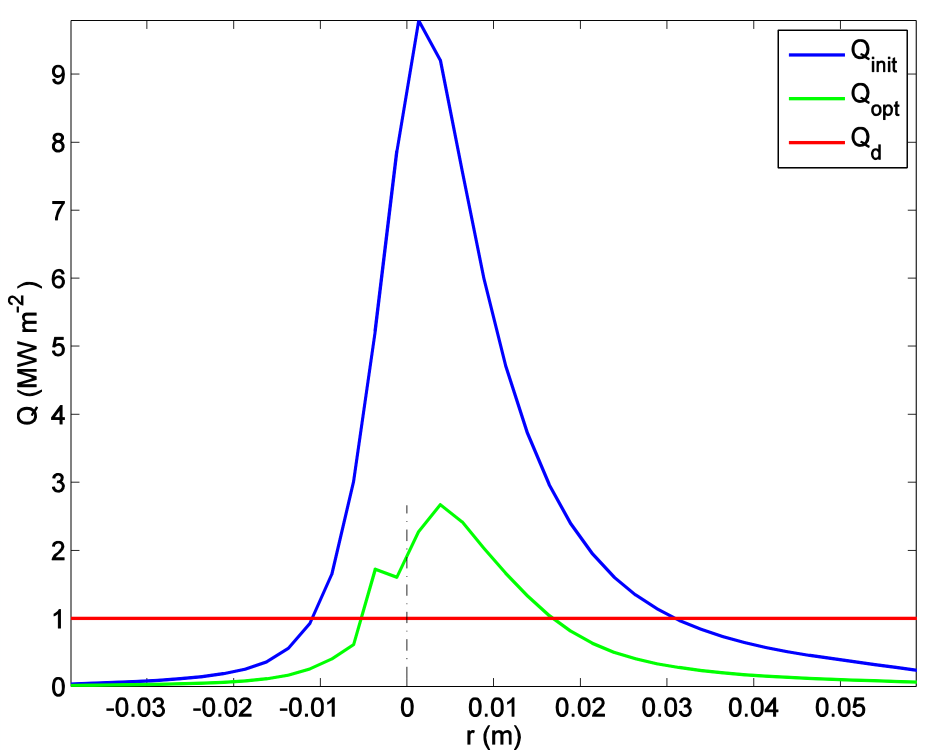
Initial configuration





Optimized Target Profile





⇒ V-shaped divertor targets

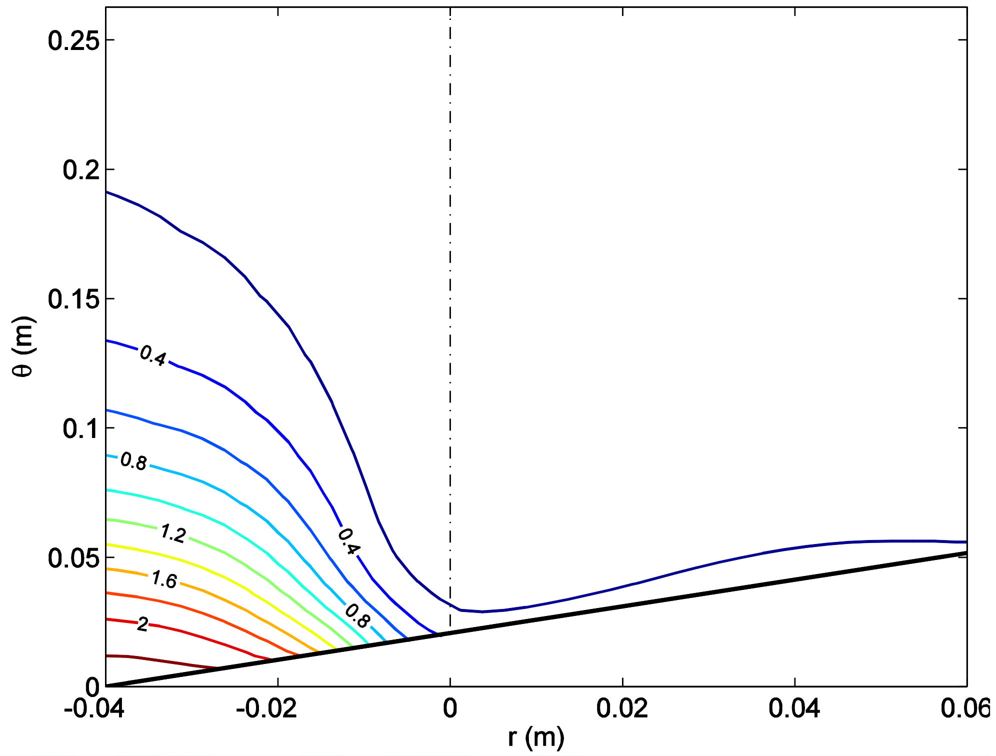
Interpretation of results

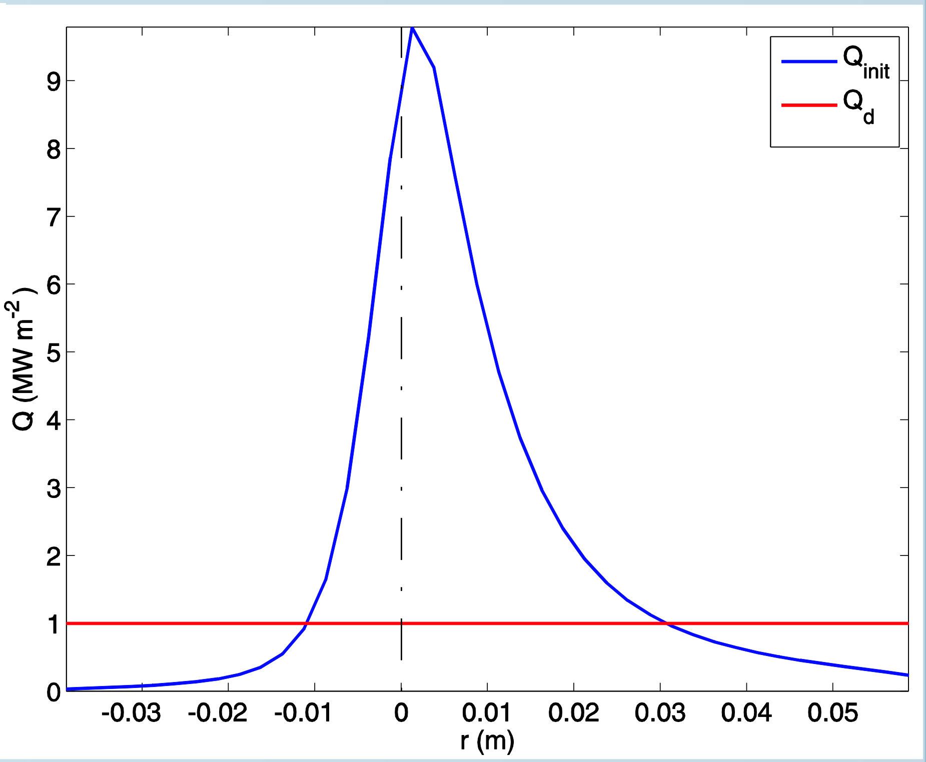
- Reduction of power flux density due to **increase in target area**

- **Shift in neutral cloud** towards the separatrix
  - Increased energy loss due to ionization
  - Increased energy loss due to impurity radiation

  $\Rightarrow$ Reduction of target temperature and power flux density

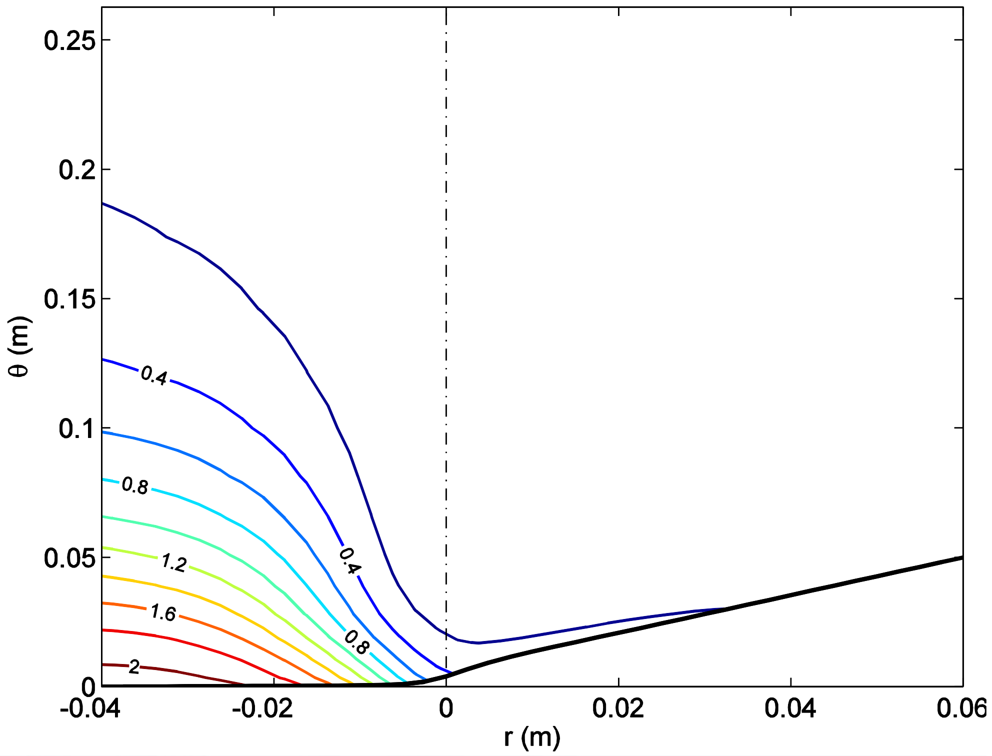
‘Time’ evolution…

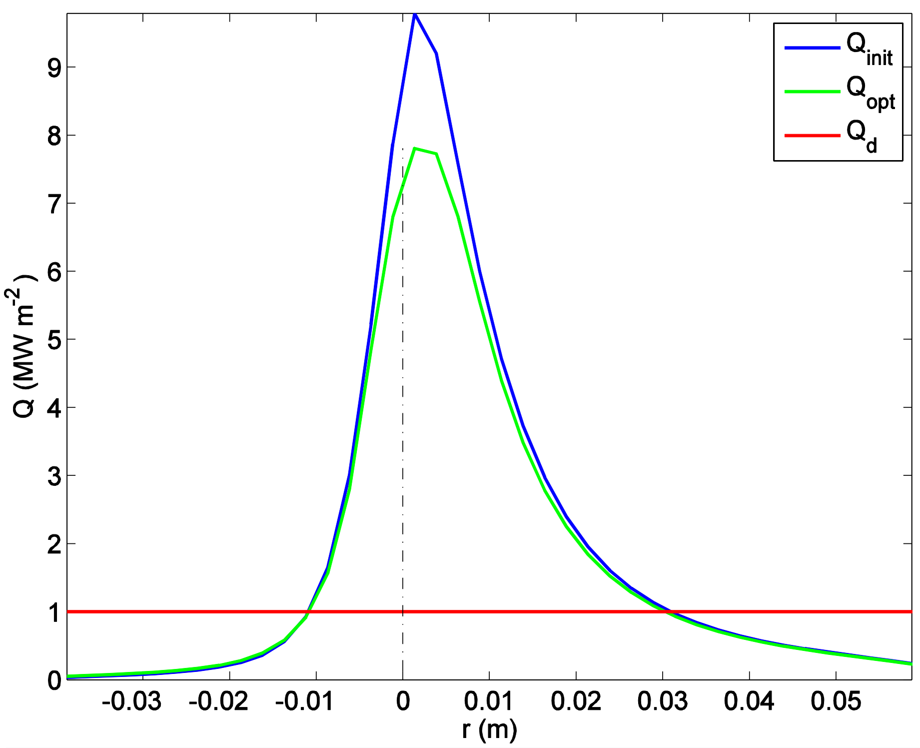




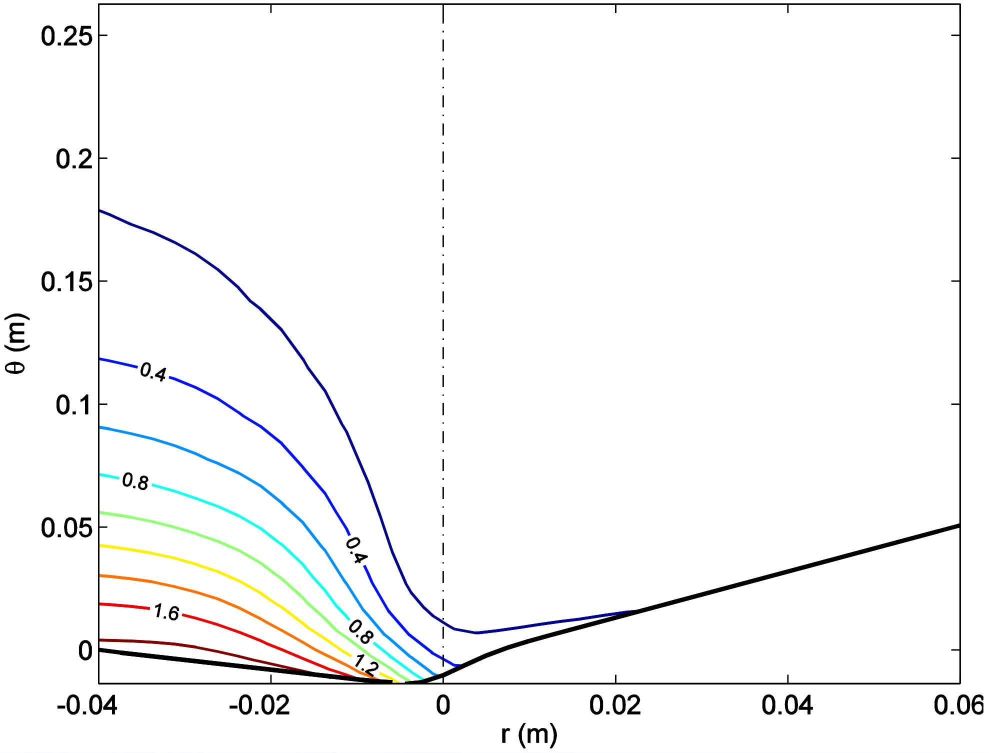
Initial configuration

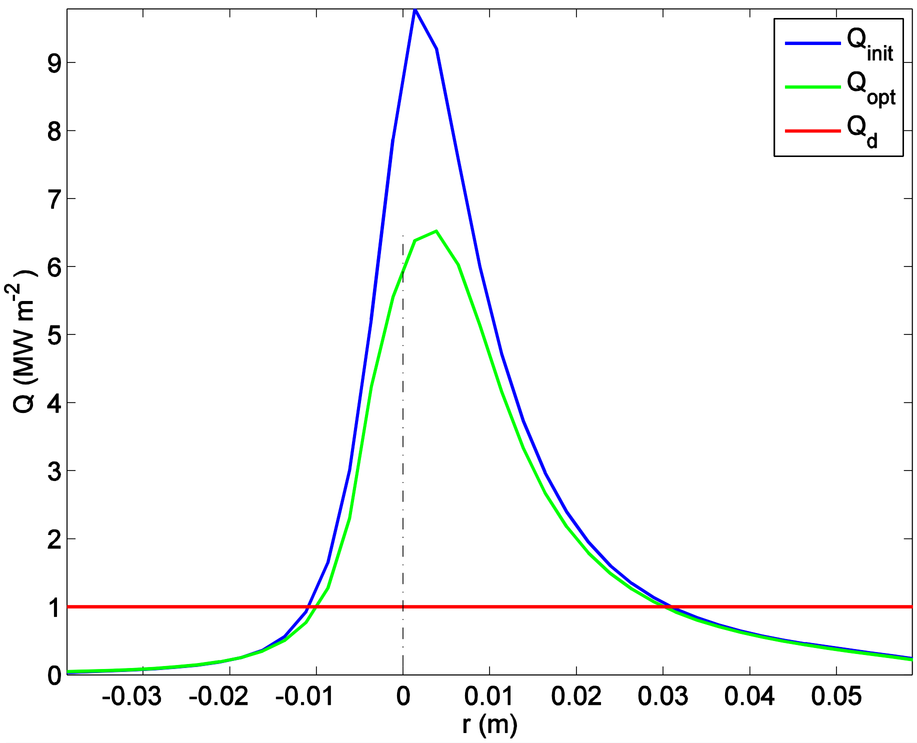
‘Time’ evolution…



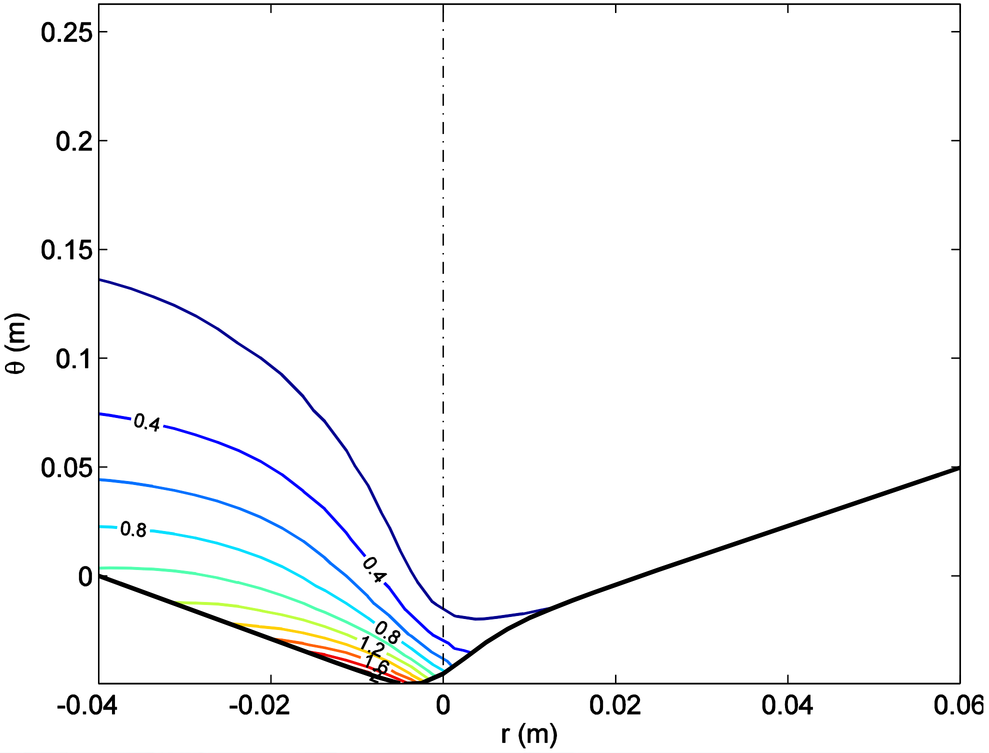


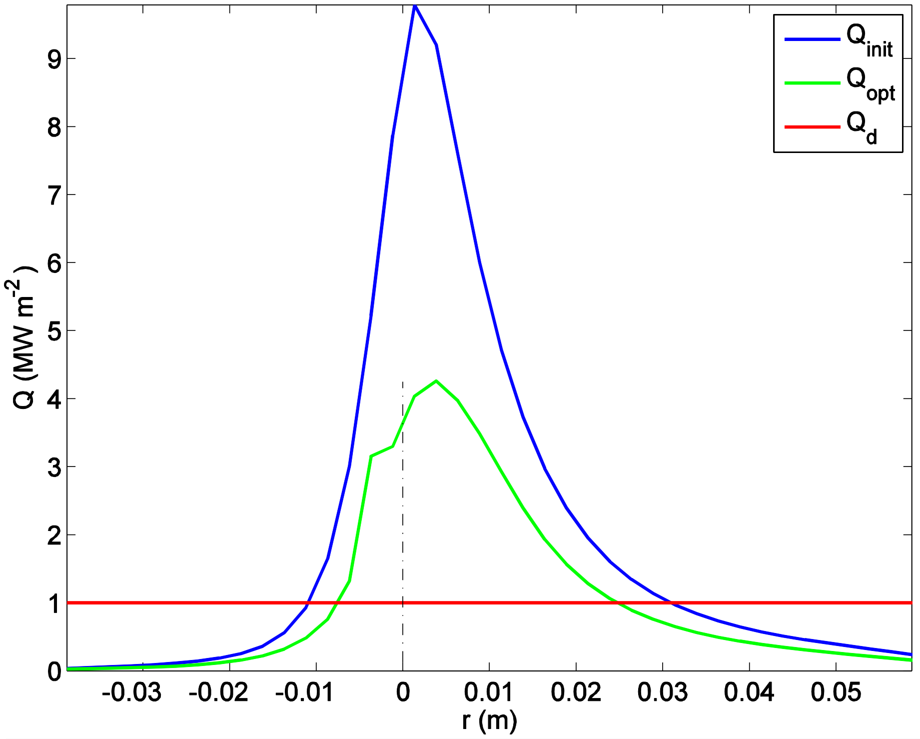
‘Time’ evolution…



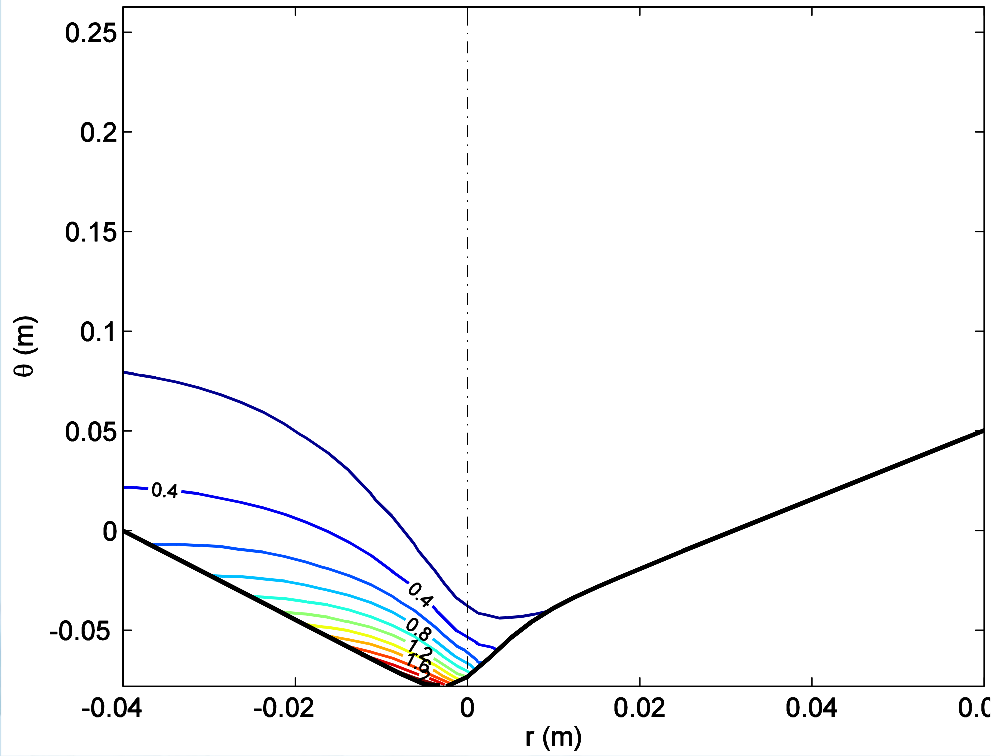


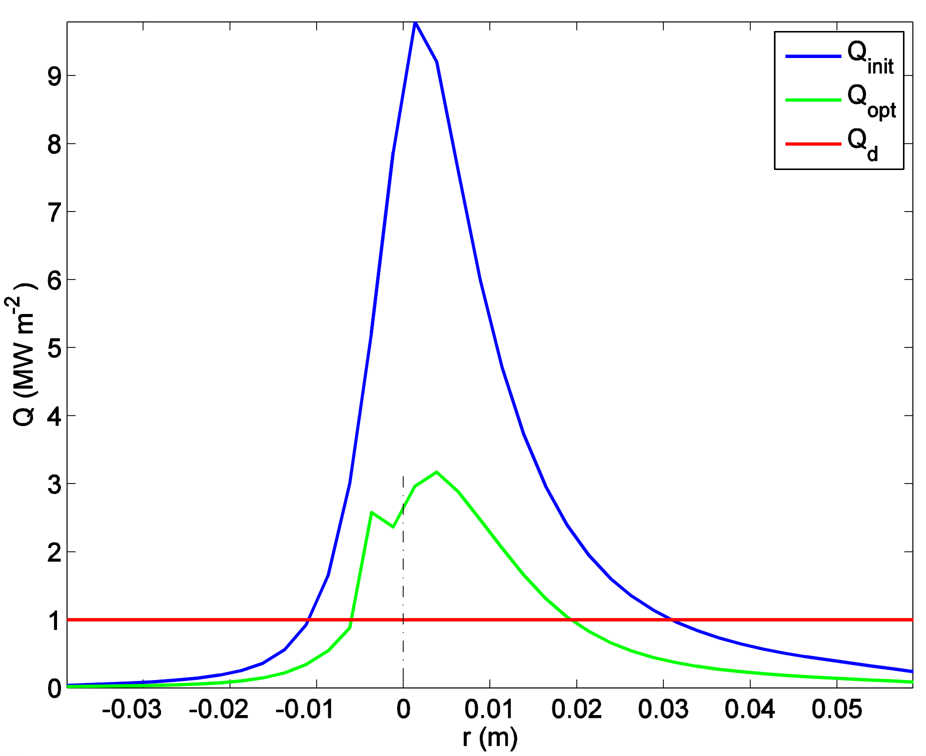
‘Time’ evolution…



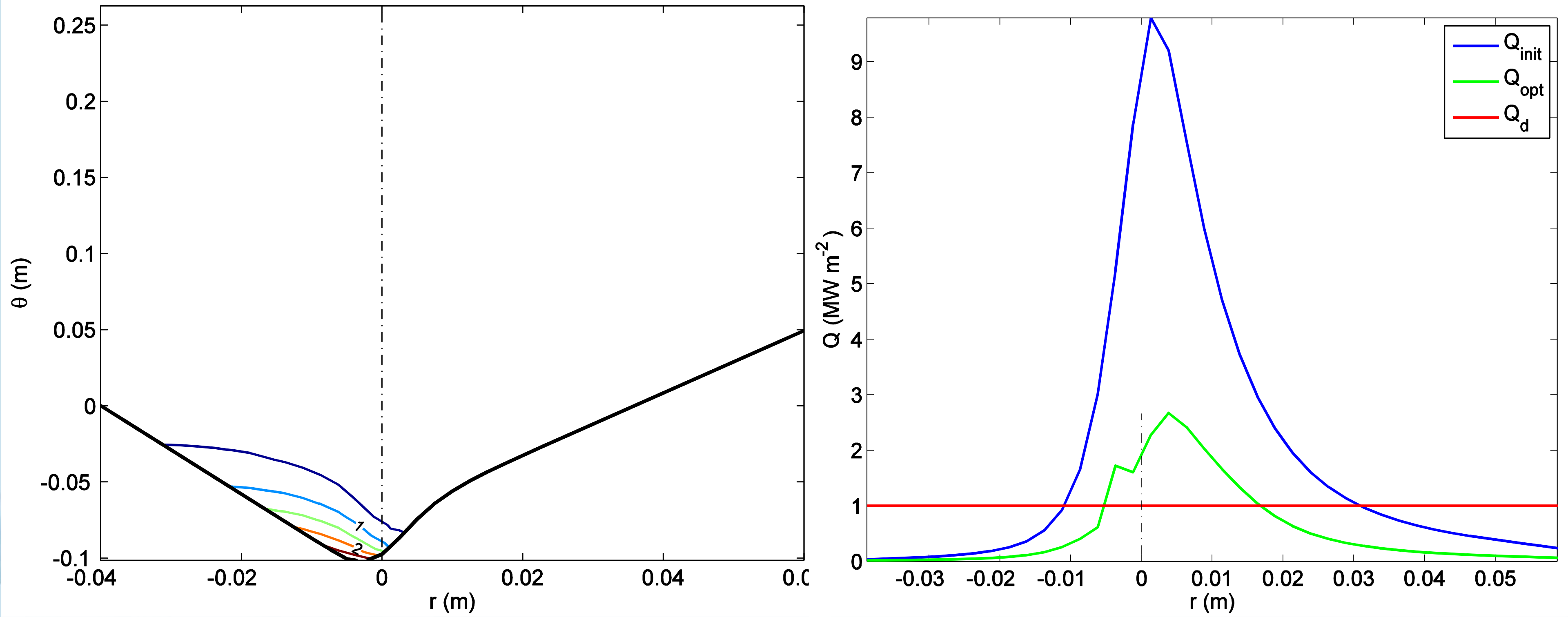


'Time' evolution…



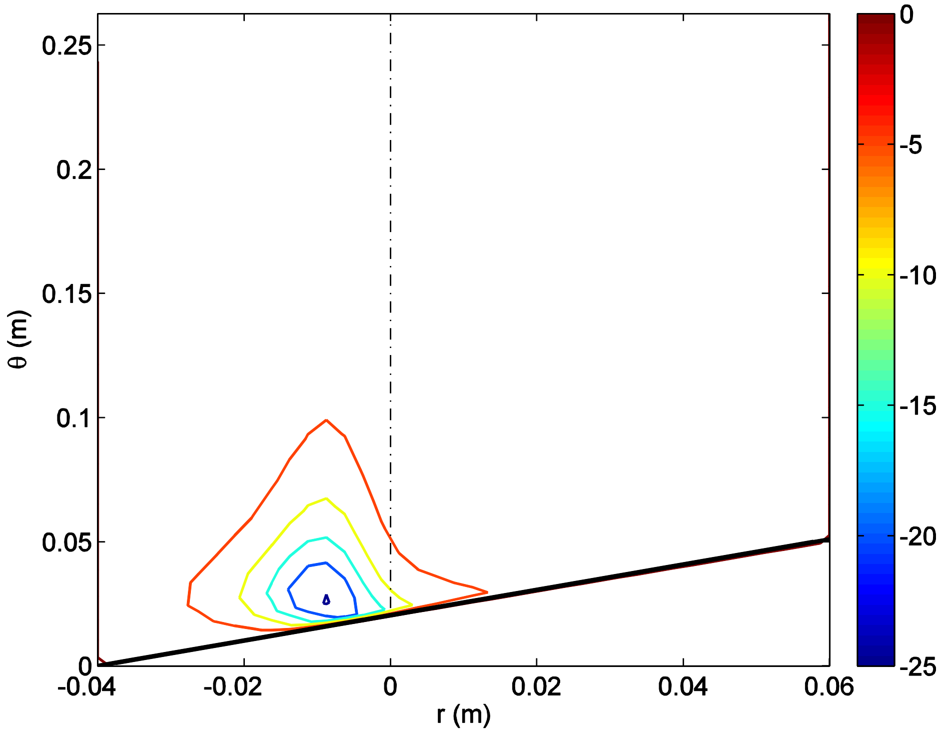


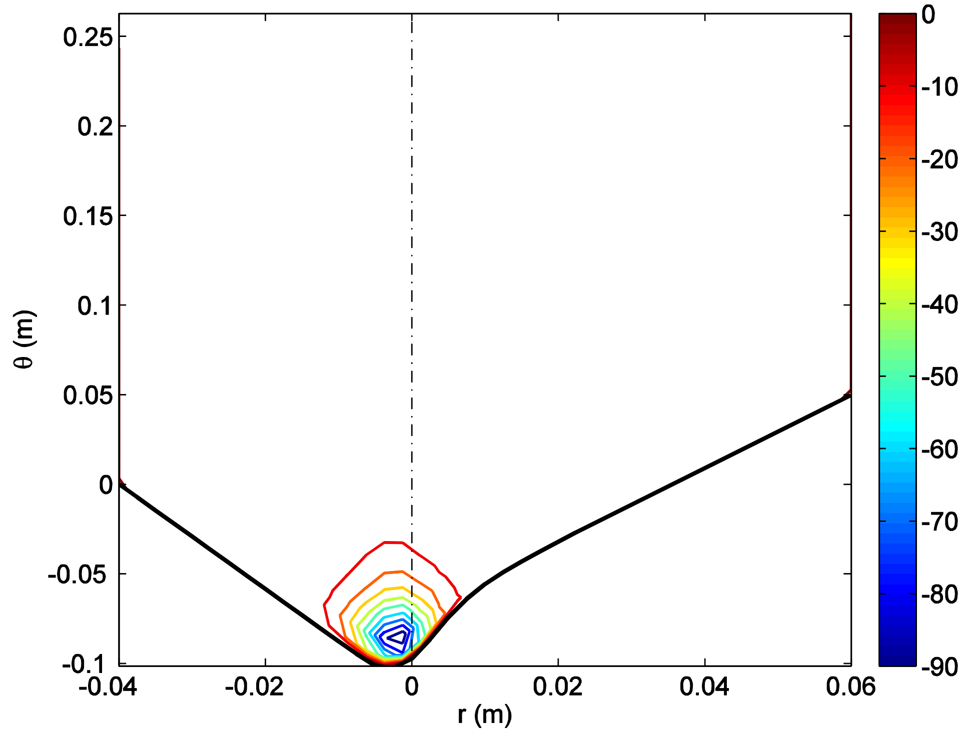
‘Time’ evolution…



Optimized

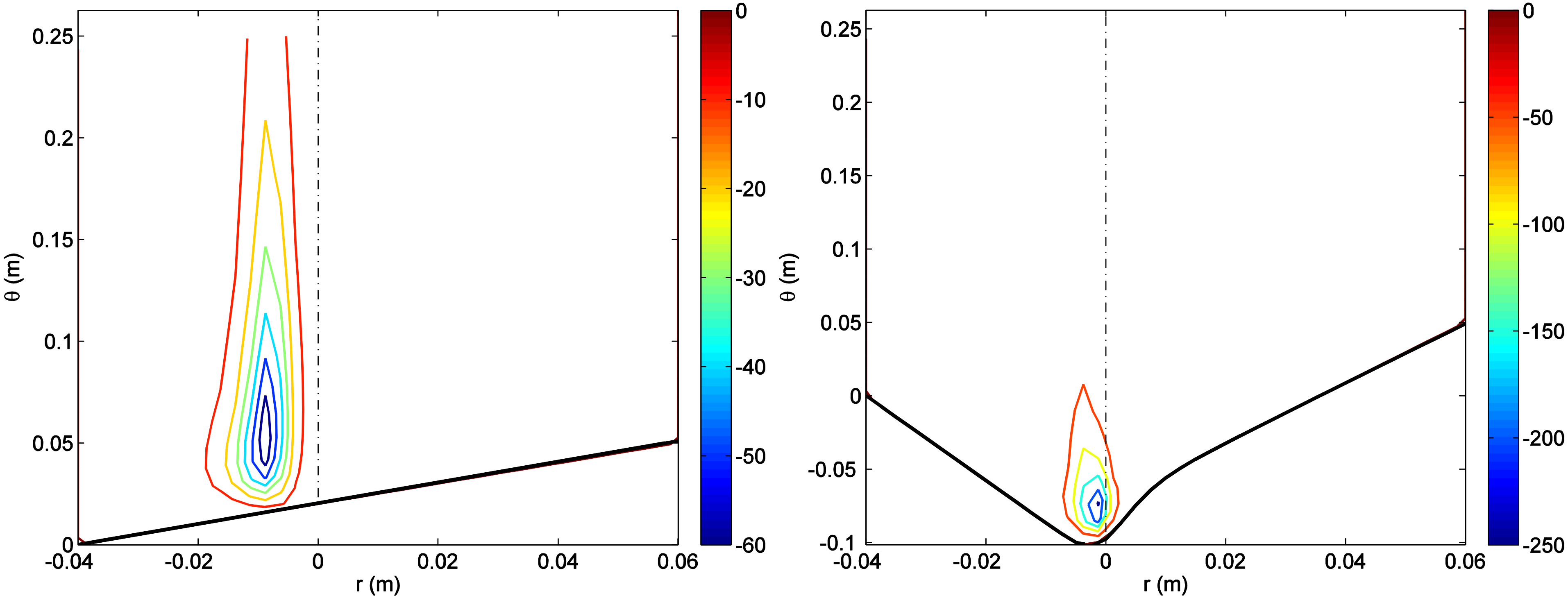
Energy sink neutral ionization (MW m^{-3})





- Shift towards separatrix
- ~5% increase total ionization energy loss

Energy sink impurity radiation (MW m^{-3})



- Shift towards separatrix
- ~10% increase total impurity radiation energy loss

Overview

- Introduction
- Target design for homogeneous power load
- Test problem
- Conclusion

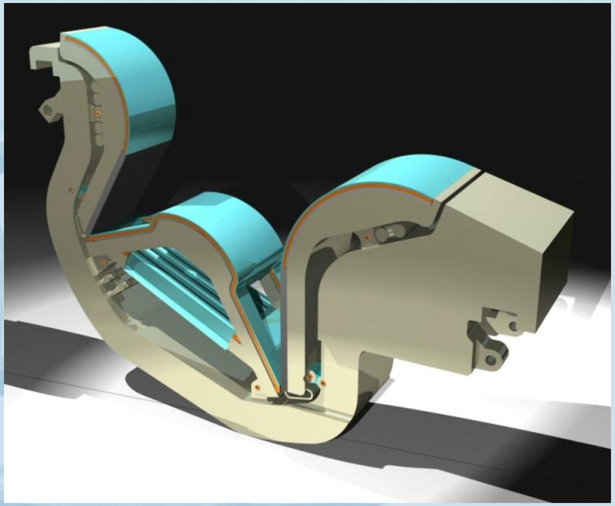
Conclusions

- A framework for optimization based divertor design is set up
- Advanced adjoint methods allow for the solution of a complete optimization cycle in an equivalent CPU time of only a few forward simulations
- Using a strongly simplified edge model, representative design features are obtained, e.g. V-shaped targets

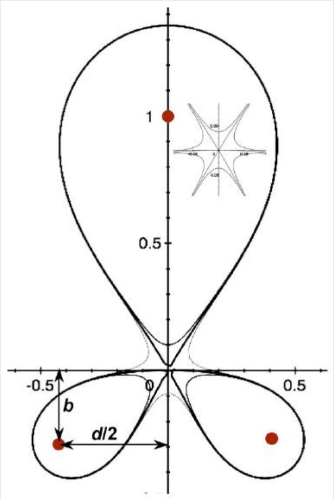
Future work...

- Divertor **shape** optimization
  - Fixed magnetic configuration
  - More complex parameterizations: shape of targets, dome, baffles,...
- Optimization of divertor **magnetic configuration**
  - Fixed divertor geometry
  - Control variables: currents through coils, location of coils,...

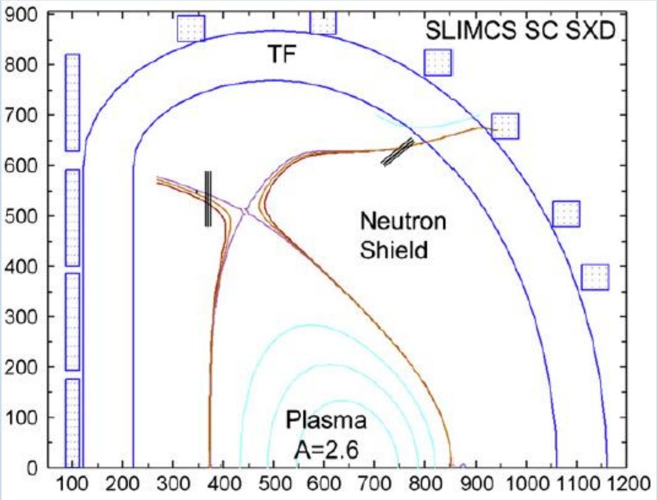
- Combined optimization
- More complete edge models



(http://www.iter.org)



(Ryutov et al., Phys. Plasmas 15, 092501 (2008).)



(Valanju et al., Fusion Eng. Des. 85, 46-52 (2010).)